A problem on maximum dense subgraphs

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Let $G = (V, E)$ be a simple connected graph. For $m = 1, 2, \ldots, |V|$ denote by $E(m)$ the maximum number of edges in an induced subgraph of $G$ of order $m$. We call this problem Edge-Isoperimetric Problem (abbr. EIP), and the vertex sets with maximum number of induced edges are called optimal sets. Further denote $\delta(m) = E(m) - E(m - 1)$ for $m = 1, 2, \ldots, |V|$, and for $k = \max_m \delta(m)$ define $\delta'(m) = k - \delta(m)$ for $m = 1, 2, \ldots, |V|$. We say that the $\delta$-sequence $\delta(1), \ldots, \delta(m)$ is symmetric if $\delta(m) = \delta'(|V| - m + 1)$ for $m = 1, \ldots, |V|$. It is easy to show that if $G$ is regular, then its $\delta$-sequence is symmetric.

**Conjecture 1**: If the $\delta$-sequence of $G$ is symmetric, then $G$ is regular.

All known to me results related to solving EIP for cartesian powers of a given graph deal with powers of regular graphs, see, e.g., [1],[2]. It would be equally interesting either to prove this conjecture or construct a counterexample. Currently, I believe in its validity.

If a counterexample is found, then what I am really interested in is to prove/disprove Conjecture 1 for graphs satisfying the following additional condition.

We say that $G$ admits nested solutions (NS) in EIP is there exists a total order on the vertex set $V$ such that for every $m = 1, 2, \ldots, |V|$ the set of the first $m$ vertices of $G$ in this order is an optimal set.

**Conjecture 2**: If $G$ admits NS in EIP and its $\delta$-sequence is symmetric, then $G$ is regular.

**References**
