Hamiltonian Paths in the Complete Graph

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Let $v$ be an odd integer and let $K_v$ be the complete graph with vertex-set $V = \{0,1,\ldots,v-1\}$. We define the length of an edge $[x,y]$ of $K_v$ as

$$l(x,y) = \min\{|x-y|, v-|x-y|\}.$$

Given an arbitrary subgraph $G$ of $K_v$, the list of edge-lengths of $G$ is

$$l(G) = \{l(x,y) : [x,y] \in E(G)\}.$$

If $L$ is a multiset consisting of $v-1$ elements, then we would like to find an Hamiltonian path using edges whose lengths are exactly the elements of $L$.

It is not hard to prove that such a path does not exist if $v$ is not a prime and $L$ is a well chosen multiset (basically if $L$ contains too many elements which are not coprime with $v$). Buratti conjectured in 2007 that this is never the case if $v$ is a prime.

**Conjecture 1 (Buratti (2007))** Given a prime $p = 2n+1$ and a multiset $L$ of $2n$ elements taken from $\{1,2,\ldots,n\}$, there exists an Hamiltonian path $H$ in $K_p$ such that $l(H) = L$.

**References**


